Tuning the dipolar interaction in quantum gases

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We have studied the tunability of the interaction between permanent dipoles in Bose-Einstein condensates. Based on time-dependent control of the anisotropy of the dipolar interaction, we show that even the very weak magnetic dipole coupling in alkali gases can be used to excite collective modes. Furthermore, we discuss how the effective dipolar coupling in a Bose-Einstein condensate can be tuned from positive to negative values and even switched off completely by fast rotation of the orientation of the dipoles.

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In the Bose-Einstein condensates (BECs) created thus far [1] atoms interact essentially only at very short distance. At low temperatures, the interaction between these neutral atoms can be typically characterized by a single parameter, the s-wave scattering length. The magnitude and sign of the s-wave scattering length can be changed by tuning the external magnetic fields in the vicinity of a so-called Feshbach resonance [2]. This technique has opened new possibilities for the study and manipulation of BECs. In particular, the value of the s-wave scattering length can be varied in real time, allowing, for example, the observation of collapsing and exploding BECs [3].

In recent years an increasing interest is devoted to dipole-dipole interactions in ultracold gases [4–12]. Dipolar interactions would largely enrich the variety of phenomena to be observed due to their long-range and anisotropic character. If atomic dipole moments are sufficiently large, the resulting dipole-dipole forces may influence, or even completely change the properties of Bose gases [4–6,11], the conditions for BCS transition in Fermi gases [7], or the phase diagram for quantum phase transitions in ultracold dipolar gases confined in optical lattices [8]. The interplay of short-range scattering and long-range interaction may give rise to phenomena like ferromagnetic order and spin waves [9]. Moreover the dipolar particles are considered to be promising candidates for the implementation of fast and robust quantum-computing schemes [10].

In this Letter, we consider the case of permanent (magnetic) dipoles. For alkali atoms with a magnetic moment m of μ_B, where μ_B is a Bohr magneton, the long range part of the magnetic dipole-dipole interaction is generally neglected since it is very small compared to the s-wave pseudo-potential. Here, we show that even in alkali BECs this dipolar interaction can be made visible if the dipole-dipole interaction is modulated by rotating the atomic dipoles. For appropriate rotation frequencies, a coupling to elementary excitations can be achieved leading to a dynamic growth of the amplitude of the excitation. In particular, we will show how the dipolar coupling in an alkali condensate can be used to excite the quadrupole mode and how it is possible to extract quantitative information about the ratio of the dipolar coupling and the usual s-wave coupling.

In ensembles of atoms with a larger magnetic moment, such as chromium where $m = 6\mu_B$, the magnetic dipole-dipole interaction energy may become comparable in size with the s-wave contribution to the mean-field energy. Therefore, even the static properties of a condensate such as the aspect ratio are significantly altered. Here, we show how the effective strength of the interaction and thus its effect on the static properties of a condensate can be varied by fast rotation of the atomic dipoles. In the case of a dipolar interaction which is stronger than the s-wave interaction only certain combinations of trap geometry and orientation of the dipoles lead to a stable (or metastable) condensate [6,4]. By tuning the dipolar interaction unstable condensates can thus be transformed into stable ones and viceversa. In this paper, we study the interaction between permanent magnetic dipoles. However, the methods we propose can also be adapted for particles with electric dipole moment such as heteronuclear molecules. In this case the dipolar interaction is expected to dominate over the s-wave interaction and thus the tuning of the interaction might be necessary to reach stability.

Manipulating the magnetic dipolar coupling in a BEC—The long-range part of the interaction between two magnetic dipole moments $\mathbf{m}_1$ and $\mathbf{m}_2$ of atoms located at $\mathbf{r}_1$ and $\mathbf{r}_2$ takes the familiar dependence

$$U_{\mathrm{dd}}(\mathbf{r}) = -\frac{\mu_0}{4\pi} \frac{3\left(\mathbf{m}_1 \cdot \hat{\mathbf{r}}\right)\left(\mathbf{m}_2 \cdot \hat{\mathbf{r}}\right) - \left(\mathbf{m}_1 \cdot \mathbf{m}_2\right)}{r^3} \qquad (1)$$

as a function of the interatomic distance $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$, where $\hat{\mathbf{r}} = \mathbf{r}/r$ and μ_0 is the magnetic permeability of the vacuum. We consider the case of a general *time-dependent* homogeneous magnetic field

$$\mathbf{B}(t) = B\,\mathbf{e}(t) , \qquad (2)$$

slowly-varying with respect to the Larmor frequency $\omega_{Larmor} = mB/\hbar$, where $m = |\mathbf{m}_1| = |\mathbf{m}_2|$ for identical atoms. A spin-polarized atomic ensembles will adiabatically follow the external magnetic field $B(t)$ and the resulting interatomic energy becomes

$$U_{\mathrm{dd}}(\mathbf{r}, t) = -\frac{\mu_0 m^2}{4\pi} \frac{3\left(\mathbf{e}(t) \cdot \hat{\mathbf{r}}\right)^2 - 1}{r^3} \qquad (3)$$

and is thus *time-dependent.*

Our analysis is based on the mean-field approach in the Thomas-Fermi limit already used in the context of dipolar Bose gases [4–12]. Such a description can be accomplished through the hydrodynamic equations for superfluids [1]:

$$\frac{\partial n}{\partial t} = -\nabla(n\mathbf{v}) , \qquad (4)$$

$$\frac{\partial \mathbf{v}}{\partial t} = -\nabla \left(\frac{v^2}{2} + \frac{\delta\mu}{M} \right) , \qquad (5)$$

where n is the density, $\mathbf{v}$ the velocity field and $\delta\mu$ the difference between the mean-field plus external fields and the chemical potential μ given by

$$\delta\mu = gn + \Phi_{\mathrm{dd}} + \frac{M\omega_0^2 r^2}{2} - \mu. \qquad (6)$$

Here M is the atomic mass, $M\omega_0^2 r^2/2$ is an isotropic harmonic potential which could be created using optical dipole forces, $g = 4\pi\hbar^2 a/M$ and a is the s-wave scattering length which we assume to be positive. $\Phi_{\mathrm{dd}}(\mathbf{r}, t)$ is the mean-field potential generated by the (time dependent) dipolar interaction

$$\Phi_{\mathrm{dd}}(\mathbf{r}, t) = \int d\mathbf{r}' \, U_{\mathrm{dd}}(\mathbf{r} - \mathbf{r}', t) \, n(\mathbf{r}') . \qquad (7)$$

In the Thomas-Fermi limit it is useful to define a dimensionless quantity

$$\varepsilon_{\mathrm{dd}} = \frac{\mu_0 m^2 M}{12\pi\hbar^2 a} , \qquad (8)$$

which is a measure of the strength of the dipole-dipole interaction relative to the s-wave scattering energy. Modifications in the ground state condensate density can only be expected if $\varepsilon_{\mathrm{dd}}$ is appreciably different from zero.

For chromium the s-wave scattering length a_{Cr} is not known. If we assume that a_{Cr} is equal to the sodium scattering length ($a = 2.8$ nm) the dipolar strength parameter becomes $\varepsilon_{\mathrm{dd}}^{\mathrm{Cr}} = 0.29$ ($m = 6\mu_B$). Therefore, the dipolar force is expected to lead to visible modification in the density of a chromium BEC. Differently, in both rubidium and sodium the magnetic dipolar energy is rather small compared to the mean-field s-wave scattering energy. The values of $\varepsilon_{\mathrm{dd}}$ are $\varepsilon_{\mathrm{dd}}^{\mathrm{Rb^{87}}} = 0.0064$ and $\varepsilon_{\mathrm{dd}}^{\mathrm{Na}} = 0.0035$, in the doubly spin-polarized ground-state, where $m = \mu_B$. For heteronuclear molecules with a permanent electric dipole moment of one Debye the corresponding dipolar coupling will be increased by approximatively the inverse square of the fine structure constant. This implies that $\varepsilon_{\mathrm{dd}}$ [13] is on the order of 10^2.

Probing the quadrupole modes in sodium and rubidium—The time modulation of the dipolar coupling (3) can be used to resonantly excite a quadrupole mode even for sodium and rubidium condensates. For simplicity we restrict ourselves to the case of a condensate in the

Thomas-Fermi regime confined in an isotropic harmonic trap with frequency ω_0 (for example $\omega_0/2\pi = 100$ Hz). Because $\varepsilon_{\mathrm{dd}}$ is small the dipole-dipole mean-field potential Φ_{dd} can be calculated using the symmetric equilibrium density distribution $n_0 = M\omega_0^2(R^2 - r^2)/2g$, where R is its radius

$$\Phi_{\mathrm{dd}}(\mathbf{r}, t) = -\frac{\varepsilon_{\mathrm{dd}}M\omega_0^2}{5} \left[3\left(\mathbf{e}(t)\cdot\hat{\mathbf{r}}\right)^2 - 1 \right] r^2 \quad r < R, \quad (9)$$

$$= -\frac{\varepsilon_{\mathrm{dd}}M\omega_0^2}{5} \left[3\left(\mathbf{e}(t)\cdot\hat{\mathbf{r}}\right)^2 - 1 \right] \frac{R^5}{r^3} \quad r > R. \quad (10)$$

The potential outside the condensate region ($r > R$) corresponds exactly to the field generated by N dipoles located in the center of the condensate. Differently, inside the condensate ($r < R$) the mean-field potential has an anisotropic but harmonic dependence.

If the interaction is driven on a resonance frequency the evolution of the condensate can be modified even for very small dipolar coupling. Consider a magnetic field rotating in the plane $x - y$ as $\mathbf{B}(t) = B\mathbf{e}(t)$, where

$$\mathbf{e}(t) = \cos(\Omega t)\hat{\mathbf{x}} + \sin(\Omega t)\hat{\mathbf{y}} . \qquad (11)$$

The resulting dipolar interatomic energy (3) and its corresponding contribution to the mean-field potential (9-10), will have a 'rotating' part [14]. For small dipolar coupling within alkali gases the non oscillating part of (9) can be neglected and the time-dependent part will resonantly couple to the 'transversal' quadrupole mode when $\Omega \approx \omega_0/\sqrt{2}$ (see Ref. [15]). After linearization of the equations of motion (4) and (5) we obtain

$$\frac{\partial^2 \delta n}{\partial t^2} = \nabla\left(\frac{gn_0}{M}\nabla(\delta n)\right) + \nabla\left(\frac{n_0}{M}\nabla(\Phi_{\mathrm{dd}})\right), \qquad (12)$$

where $\delta n = n - n_0$ is the density displacement around the equilibrium density n_0. The time-dependent part of Φ_{dd} corresponds in Eq. (12) to the rotation of an anisotropic harmonic trap with deformation

$$\varepsilon = \frac{\omega_X^2 - \omega_Y^2}{\omega_X^2 + \omega_Y^2} = \frac{3}{10}\varepsilon_{\mathrm{dd}} , \qquad (13)$$

where ω_X and ω_Y are the maximum and minimum values of the rotating trap frequencies. Neglecting the time-independent part of (9) the density will evolve as

$$\delta n = -\frac{3\,\varepsilon_{\mathrm{dd}}M\omega_0^2}{10\,g} r^2 \sin^2\theta \sin(2\phi - 2\Omega t)\,\Omega t, \qquad (14)$$

where ϕ is the azimuthal angle ($r < R$). It is noteworthy that the rotating part [14] of (9) couples *only* the first quadrupole mode in the Thomas-Fermi limit [15]. From the study of the evolution of the condensate deformation

$$\alpha = \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle} = -\frac{3}{5}\varepsilon_{\mathrm{dd}}\,\Omega t\,\sin(2\Omega t) \qquad (15)$$

we can extract information about the ratio of the dipolar coupling versus the standard mean-field energy. If the
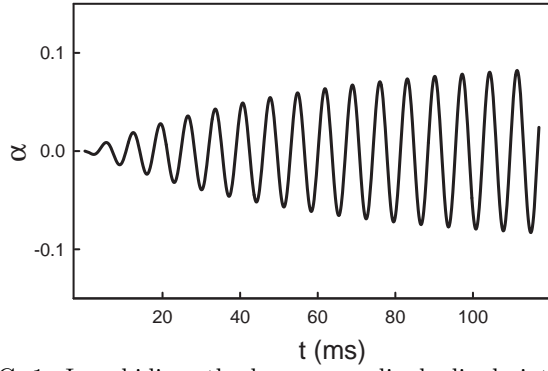
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FIG. 1. In rubidium the long-range dipole-dipole interaction does not modify in an appreciable way the ground state density. When the magnetic dipoles are driven on resonance with a transversal quadrupole mode the anisotropy α (Eq. (15)) is growing with time (here we assume a quality factor $Q = 50$).
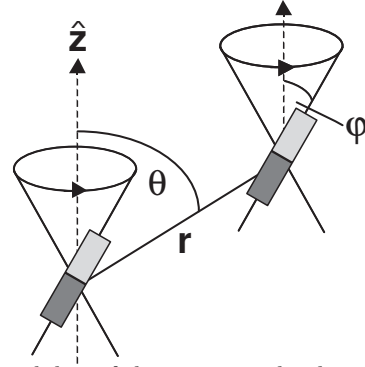

FIG. 2. Tunability of the magnetic dipole interaction. The atomic dipoles follow adiabatically the total magnetic field (16) when the field is sufficiently strong. The angle φ between the precessing total magnetic field and the z-axis characterizes the interaction. $\varphi = 0$: the magnetic dipoles are simply polarized in the z direction. $\varphi = \pi/2$: the magnetic dipoles follow the rotating magnetic field and as a result of the fast rotation the averaged interaction becomes $-1/2$ times the interaction in the polarized case. $\varphi = 54.7^o$ (magic-angle): the dipolar interaction averages to zero.

quality factor $Q = \sqrt{2}\omega_0\tau$ of such a resonance, where τ is the corresponding decay time, is assumed to be 50 [16] the amplitude of the oscillation of the condensate anisotropy α is expected to be $|\alpha| = 3Q\varepsilon_{dd}/10 \sim 9.5 \times 10^{-2}$ for rubidium (see Fig. 1). We have performed our calculation in the linear regime and for an isotropic trap. The non-linearity of the mean-field is expected to give a significant correction for larger Q. The extension of the calculation to an axially symmetric cigar-shaped trap is expected to yield similar results.

Tunability of the magnetic dipolar interaction: — We consider a condensate in an axially symmetric harmonic trap, with radial and axial trap frequencies ω_r and ω_z. The magnetic field

$$\mathbf{B}(t) = B\left[\cos(\varphi)\hat{\mathbf{z}} + \sin(\varphi)\left(\cos(\Omega t)\hat{\mathbf{x}} + \sin(\Omega t)\hat{\mathbf{y}}\right)\right] \quad (16)$$

is a combination of a static magnetic field B_z directed along the z direction and a fast rotating field B_ρ in the radial plane. The frequency is chosen such that the atoms are not significantly moving during the time Ω^{-1}; while the magnetic moments will follow adiabatically the external field $\mathbf{B}(\mathbf{t})$; this corresponds to $\omega_{Larmor} \gg \Omega \gg \omega_{trap}$, where ω_{trap} is any of the external trap frequencies. In this limit we can consider the average of the interaction (3) in the period $2\pi/\Omega$, resulting in the cylindrically symmetric interatomic potential

$$\langle U_{dd}(\vec{r})\rangle = -\frac{\mu_0 m^2}{4\pi}\left(\frac{3\cos^2\varphi - 1}{2}\right)\left(3\cos^2\theta - 1\right). \quad (17)$$

The averaged interaction energy (17) equals $(3\cos^2\varphi - 1)/2$ times the interatomic energy of dipoles aligned along the z-axes. Note that this factor can be changed continuously from 1 to $-1/2$, providing the possibility to change the dipolar interaction from attractive to repulsive. At a particular angle $\varphi_M = 54.7^o$ the dipolar interaction averages to zero. This angle, the so-called magic-angle, is well known in solid-state nuclear magnetic resonance

(NMR) technology [17]. The tunability of the dipolar interaction will allow to observe changes in the condensate eccentricity which should be a 10% effect in Cr assuming $a_{Cr} = a_{Na}$ or in the collective excitation frequencies [4,18].

Stabilizing strongly interacting dipolar gases:—Let us now apply this tunability to the stability diagram of dipolar BECs. We consider the case in which the dipolar coupling is larger than the s-wave pseudopotential ($\varepsilon_{dd} \gtrsim 1$). Here the tunability can stabilize strongly interacting dipolar gases like heteronuclear molecules which otherwise exhibit instability over a wide range of parameters. This regime, considered also in Ref. [4,6], may be achieved in a cloud of magnetic dipolar atoms, by making use of a Feshbach resonance to reduce the s-wave scattering length.

In a spin polarized cloud the stability diagram as well as the condensate anisotropy can be studied in the Thomas-Fermi limit as function of ϵ_{dd} (see Ref. [4–6]). When $\epsilon_{dd} \geq 1$ the cloud may be unstable against collapse, i.e. can be always possible to find a density configuration of arbitrary large negative energy. The stability as ϵ_{dd} and the trap anisotropy $l = \omega_z/\omega_r$ are varied can be investigated in analogy to Ref. [4].

Within the Thomas-Fermi approximation, the stability analysis can be made using a scaling variational approach [19]. Assuming a density for the condensate of the form

$$n(x, y, z) = \frac{1}{\lambda_r^2\lambda_z}n_0\left(\frac{x}{\lambda_r}, \frac{y}{\lambda_r}, \frac{z}{\lambda_z}\right), \quad (18)$$

where λ_r and λ_z are scaling variational parameters and n_0 is a spherically symmetric density distribution (for instance the usual Thomas-Fermi inverted parabola). The condensate anisotropy λ_r/λ_z can be found by minimiz-
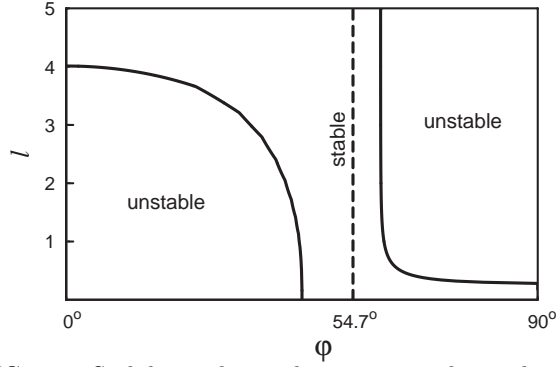
FIG. 3. Stability phase diagram in the φ-l plane ($l = \omega_z/\omega_r$) obtained in the Thomas-Fermi limit ($N \to \infty$) and with $\varepsilon_{dd} = 4$. Condensate are always stable in the vicinity of the magic angle $\varphi_M = 54.7^o$.

ing the expectation value of the total energy $H_{tot} = H_{ho} + H_{int}$ where

$$H_{ho} = c_1 N m \left(2\omega_r^2 \lambda_r^2 + \omega_z^2 \lambda_z^2 \right) \tag{19}$$

is the harmonic potential energy and

$$H_{int} = c_2 \frac{N^2 g}{\lambda_r^2 \lambda_z} \left[1 - \varepsilon_{dd} \left(\frac{3\cos^2\varphi - 1}{2} \right) f \left(\frac{\lambda_r}{\lambda_z} \right) \right] \tag{20}$$

the mean-field interaction energy, where c_1 and c_2 are numerical constants [20] and

$$f(\kappa) = \frac{1 + 2\kappa^2}{1 - \kappa^2} - \frac{3\kappa^2 \tanh^{-1}\sqrt{1 - \kappa^2}}{(1 - \kappa^2)^{3/2}} \tag{21}$$

derives from an angular integration.

The stability can be varied for a fixed value of ε_{dd} and the trap anisotropy l just varying one easy experimental parameter, namely the angle φ. In Fig. 3 we show one example of how the stability phase diagram for $\varepsilon_{dd} = 4$ can be varied as function of the angle φ. For very large ε_{dd} the stable region in the φ-l plane becomes a thin region around the magic angle line.

Conclusion—We have shown that the anisotropy of the dipole-dipole interaction of a polarized gas can be used to tune the strength of the dipolar coupling and to excite collective excitations. Tuning the dipolar coupling will become an important tool for designing atomic quantum gases with novel properties and dynamical aspects. This approach in optically trapped Bose condensed gases should allow to make even very weak dipolar coupling visible with experimentally feasible parameters (like B fields in the Gauss region and spinning frequencies up to several 10 kHz). The tuning of the interaction might become a relevant technique for stabilizing strongly interacting dipolar systems like heteronuclear molecular gases.

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[14] The angular dependence of (9) and (10) $\left[3 \left(\mathbf{e}(t) \cdot \hat{\mathbf{r}} \right)^2 - 1 \right]$ for $\mathbf{e}$ given by (11) can be rewritten in terms of spherical harmonics as $12\sqrt{2\pi/15}\mathrm{Re}[Y_{22}e^{-i2\Omega t}] - \sqrt{4\pi/5}Y_{20}$. Because of the r^2 dependence in (9) only the first quadrupole mode in the Thomas-Fermi limit (with n=0 number of radial nodes in the density displacement δn) [15] is resonantly excited.
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